**Exercise II.22.6** : Here is an example of exercise where we feel that we will be able to solved it simply by applying Thales's and Pythagoras's theorems several times, but that is not the case!

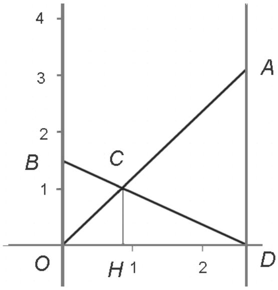


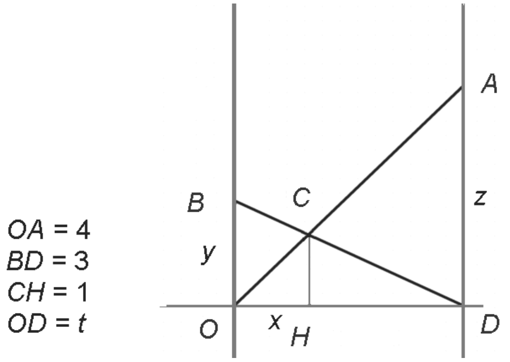

Figure II.22.11 : Two ladders of respectively 3 m and 4 m are positioned as shown across a corridor. Knowing that they cross at one meter above the ground, we want to calculate the width of the corridor.

The available data are :
$OA = 4$ m
$BD = 3$ m
$CH = 1$ m
Show that $OD = 2.603...$ m.

*Hint* : By writing down the various geometric relationships we have, we arrive at a 4th-degree equation for $OH$, and we solve it numerically as we have already done many times before.

*Solution* : Let's introduce a few quantities that are currently unknown.

- $x = OH$
- $y = OB$
- $z = DA$
- $t = OD$




In triangle $ODA$, let's apply Thales's theorem. This yields :

$$\frac{1}{x} = \frac{z}{t}$$

or

$$t = xz \tag{1}$$

And by Pythagoras, we also have :

$$t^2 + z^2 = 16 \tag{2}$$

Similarly, in triangle $ODB$, by Thales we have :

$$\frac{1}{t-x} = \frac{y}{t}$$

or

$$t(y-1) = xy \tag{3}$$

And by Pythagoras, we also have :

$$t^2 + y^2 = 9 \tag{4}$$

Now we are going to do some algebra (= "turn the crank") to arrive at an equation that acts as a constraint on a single variable.

We will use equations (1) to (4) – which synthesize everything we know – to perform eliminations and reach an equation with one unknown.

Observe that we have four equations for four unknowns, so the situation looks a priori well engaged.

First, we can eliminate $t$ and arrive at :

$$\begin{cases} x^2z^2 + z^2 = 16 \\ z(y-1) = y \\ x^2z^2 + y^2 = 9 \end{cases}$$

Then we can eliminate $x$ by subtracting the third equation from the first. We will calculate $x$ afterwards. We obtain :

$$\begin{cases} z^2 - y^2 = 7 \\ z(y-1) = y \end{cases}$$

Finally, we obtain an equation with one unknown in $y$ :

$$\frac{y^2}{(y-1)^2} - y^2 = 7$$

This gives :

$$y^4 - 2y^3 + 7y^2 - 14y + 7 = 0$$

This is a 4th-degree polynomial equation with one unknown. Even if Ludovico Ferrari (1522–1565) showed how to solve them with exact algebraic formulas, we are not going to do like him. We will look for the value of $y$ by approximation. We know that $y$ is somewhere between 1 and 2. So let's try different values.

We start looking between 1 and 2 with a step of 0.1 :

| | |
|---|---|
| step | 0.1 |
| | |
| y | value of polynomial |
| | |
| 1.0 | -1.0000 |
| 1.1 | -1.1279 |
| 1.2 | -1.1024 |
| 1.3 | -0.9079 |
| 1.4 | -0.5264 |
| 1.5 | 0.0625 |
| 1.6 | 0.8816 |
| 1.7 | 1.9561 |
| 1.8 | 3.3136 |
| 1.9 | 4.9841 |
| 2.0 | 7.0000 |

We see that $y$ is between 1.4 and 1.5. Let's try between these two values with a step of 0.01 :

| | |
|---|---|
| step | 0.01 |
| | |
| y | value of polynomial |
| | |
| 1.40 | -0.5264 |
| 1.41 | -0.4772 |
| 1.42 | -0.4259 |
| 1.43 | -0.3725 |
| 1.44 | -0.3170 |
| 1.45 | -0.2592 |
| 1.46 | -0.1994 |
| 1.47 | -0.1373 |
| 1.48 | -0.0729 |
| 1.49 | -0.0064 |
| 1.50 | 0.0625 |

The unknown value $y$ is between 1.49 and 1.50. By continuing a bit, we arrive at :

$$y \approx 1.490935$$

We deduce all the other unknowns.
First :

$$z = \frac{y}{y-1} \approx 3.036930$$

Then :

$$x = \sqrt{\frac{16-z^2}{z^2}} \approx 0.857207$$

And :

$$t = xz \approx 2.603279$$

**Epilogue** : We did indeed only use Thales and Pythagoras. But this did not lead us only to the second-degree trinomial equations with which we are familiar. It led us to a 4th-degree equation with one variable (in this case $y$, but we could have built one with another variable).

We used our favorite method to solve any equation, even when it is difficult or even impossible by algebra : we tried a collection of values to find approximately its root, which would lead us to the solution of our problem.

Once we had $y$, it was immediate to find the other unknowns.

Note that our 4th-degree polynomial in $y$ has another real root around 0.73301, but we know that it cannot be the value of $y$ that we are looking for.

This is the opportunity to recall one of the great messages of all our math books : the essence of mathematics is not the resolution of equations using impressive calculations. That is only one technique among others at one stage of the resolution of our problem.

We have even said that mathematics is only the more or less abstract domain in which we arrive when we have mathematized our real-life problem.

The scientist's job is first to understand and formulate their real problem well in order to be able to mathematize it. This mathematization is, in our opinion, the most important step for the scientist who uses mathematics. Then it's just menial chores :-)

School focuses too much on mathematical *techniques* and not enough on the *mathematization* of real-life problems (for those that lend themselves to mathematization).

A century ago, in primary school of the old days when there were crossing trains and filling cisterns, the problems set for schoolchildren were closer to real problems. The pupil had to mathematize them to solve them. It was more significant and instructive than asking : give the four roots of the polynomial

$$y^4 - 2y^3 + 7y^2 - 14y + 7 = 0$$

or, show that it has two real roots, and still a trinomial without a real root (we call such a trinomial an irreducible trinomial).

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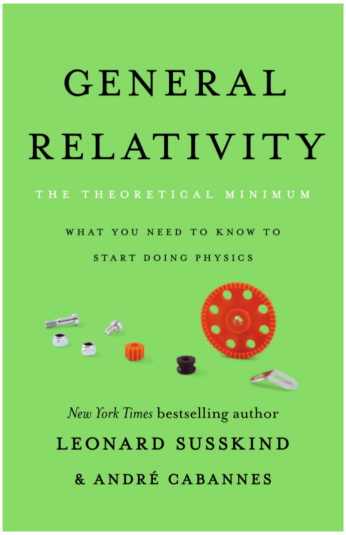


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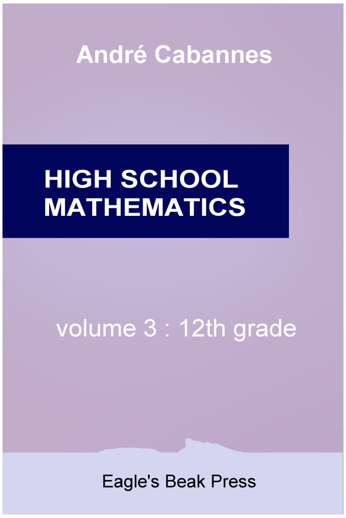


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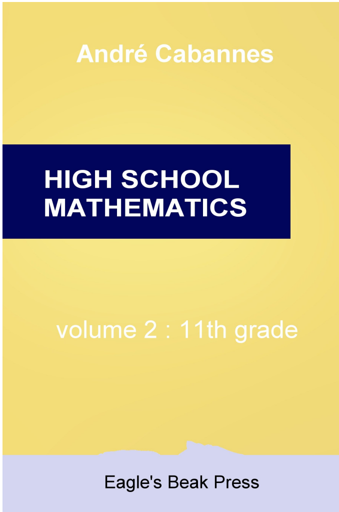


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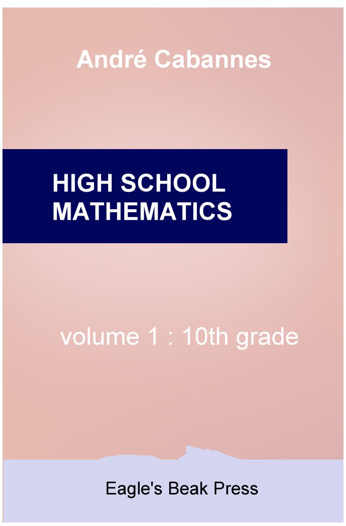


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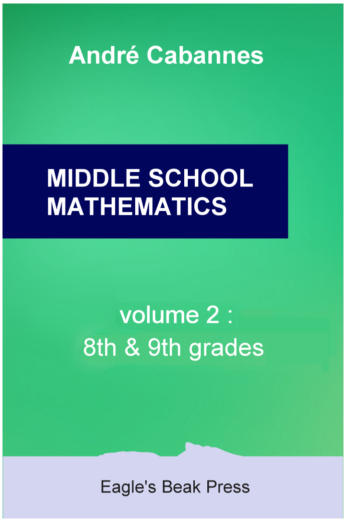


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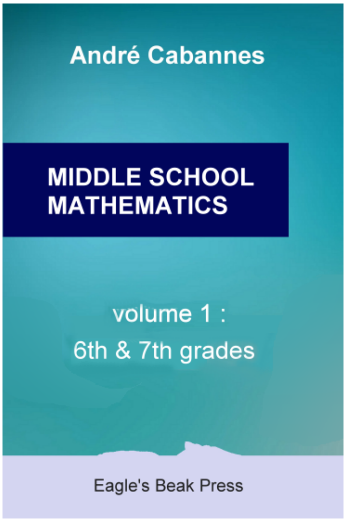


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